CONCERNING A GRAPHICAL DEVICE FOR PRESSURE REDUCTION.

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SYNOPSIS.

This paper is essentially an expansion and elaboration of an article by Schwerdt and Loebe on a graphical solution of the hypsometric formula. The steps from the forming of a linear equation and the substitution of terms from the hypsometric equation, to the final drawing of the nomogram are traced. The device enables one to determine by means of a straight line joining two points representing known elements of the hypsometric equation, the value of a third element. These elements are: (1) Sea-level pressure, (2) pressure at some level above sea level, and (3) the mean temperature of the air column. There are tables for the construction of a chart, and a reproduction of the completed chart for reducing pressures within the lowest two kilometers of the atmosphere.

One of the principal arguments against the use of graphical methods as aids in making scientific computations is their inaccuracy, for often it is not possible, either because of the nature of the fundamental formulæ or because of the difficulty of accurately drawing the chart, to secure the desired refinement. On the other hand, a nomogram that is an accurate representation of a formula and one that possesses the merits of being correctly drawn and not too complicated, makes a direct appeal. Meteorologists, and others, who have had occasion to make pressure reductions by means of the Laplacian hypsometric formula for special conditions, or for places for which reduction tables have not been constructed, are unanimous in their expressions of opinion regarding the labor and tedium of the process. It is with special interest, therefore, that we have read the article by Hans G. Schwerdt and W. W. Loebe, entitled *Eine nomographische Tafel zur Luftdruckreduktion*,[1] wherein is presented a solution of the problem of graphical pressure reduction.

The object of these authors is to produce a graph of such simplicity that upon joining two given points with a straight line, a third point on that line will yield information regarding an unknown element. For example, if one knows the station pressure and the temperature argument at a given place, it is possible, by producing the line joining the two points which represent these two elements until it intersects a scale which represents pressure at sea level, to read off the pressure reduced to sea level directly; or one can read off directly pressures at any level in the free air, if the temperature argument and station pressure are known; or (as might occur in some studies) if the pressure is known at two levels in the atmosphere, the projected line will give the mean temperature of the air column.

The mathematical explanation of the drawing, as presented by the authors, is hardly satisfying, because it does not generalize the method sufficiently, and because it possesses so much of the Laplacian *il est facile de voir*. An attempt is made in this paper, therefore, to point out the processes which were employed by the authors and to present general formulæ for preparing the chart.

The hypsometric formula of Laplace may be stated as follows:

$$\mathrm{Log}\, B - \log b - h/[18387\ (1+\alpha t)] = 0,$$

or, in a manner more convenient for our purpose,

$$\mathrm{Log}\, B - \log B_0 - \log b + \log b_0 - 0.0000543h/(1+\alpha t) = 0 \quad (1)$$

in which B is sea-level pressure, b is station pressure, h the length of the air column, t the mean temperature of the air column, α a constant, 0.00367, and B_0 and b_0 two pressures which are of use in preparing the diagram and which will be discussed later, it being mathematically obvious that B_0 must equal b_0.

If we erect a set of Cartesian coordinate axes, and desire to have a straight line serve us in determining pressures, it is clear that we must have three points—(x_1, y_1) being a point on the scale representing the mean temperature of the air column; (x_2, y_2) being a point on a scale representing station pressure; and (x_3, y_3) a point on a scale representing the pressure at the reduction level. The condition that a straight line shall pass through these points is

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

or,

$$x_2 - y_2 x_3/y_3 - x_1\ (1 - y_2/y_3) = 0 \quad (2)$$

It remains, now, to reconcile equations (*1*) and (*2*).

In the above paragraph specific duties were assigned to the three points—it is now necessary to assign them their positions on the chart. (x_1, y_1) is to take care of the temperature argument, and, for convenience, we will place it on the x axis. That is to say, $y_1 = 0$, and x_1 is that portion of the formula dealing with temperature, or $1/(1+\alpha t)$. Let the scale representing sea-level pressures be parallel to the x axis and some distance c below it. Thus, x_2 will care for the sea-level pressures and will be equal to $\log B - \log B_0$. The station-pressure scales are located between the two scales, $y_1 = 0$ and $y_2 = c$; and their distance from the x axis will be a function of the height of the station above sea-level. This is expressed by y_3, and from (*1*) and (*2*) it is found that

$$\begin{aligned} y_3 &= y_2/(1+0.0000543h) \\ &= c\ /(1+0.0000543h) \end{aligned}$$

In a similar manner we find that

$$-y_2 x_3/y_3 = -\log b + \log b_0$$

or

$$x_3 = y_3(\log b - \log b_0)/c$$

Now, let us summarize the expressions for the various scales:

$x_1 = 1/(1+\alpha t)$	$y_1 = 0$
$x_2 = \log B - \log B_0$	$y_2 = c$
$x_3 = y_3(\log b - \log b_0)/c$	$y_3 = c/(1+0.0000543h)$

These equations give the coordinates of the three points on the straight line; but if the numerical values for the various terms were chosen at random, it might easily follow that the graph would not be usable because of the inconvenience of the scales. For instance, it is necessary to expand the horizontal scales in order to make the units large enough to read with the required precision. For that reason, the horizontal scales must be magnified by a certain factor k which may be assigned any value that may prove convenient for each of the different uses of the table. If the values of x are thus magnified, we have:

$$x_1 = k/(1+\alpha t),\ x_2 = k(\log B - \log B_0),\ \text{and}\ x_3 = k y_3(\log b - \log b_0)/c.$$

[1] *Meteorologische Zeitschrift*, May, 1921, pp. 139-142.

The authors mentioned above chose to magnify the horizontal scales by the factor 4,000 and to make the value of c, or the vertical extent of the diagram, 200 units. When they did this, they found that the distribution of the x_3 scales is better if, instead of making $x_1 = 4{,}000/(1+\alpha t)$ and $y_3 = c/(1+0.0000543h)$, the factor 10 is taken from the denominator of y_3 and placed in the denominator of x_1. This can be done, as is obvious from (*1*). With these changes effected, the values of the coordinates become:

$$\begin{array}{l|l} x_1 = -400/(1+\alpha t) & y_1 = 0 \\ x_2 = 4{,}000 \log B - 4{,}000 \log B_0 & y_2 = -200 \\ x_3 = -20 y_3 (\log b - \log b_0) & y_3 = -200/(1+0.000543h) \end{array}$$

Now, what are to be the values of B_0 and b_0? It is clear that the more nearly perpendicular to the scales is the line on which the three points lie, the greater will be the convenience and accuracy of the chart. It is desirable, therefore, to have the middle of the temperature scale nearly above the most frequent value of sea-level pressure. If it is decided to locate the point of 0° C. above 760 mm. pressure, one may proceed as follows: If $t = 0°$ C., $x_1 = -400$. This is to be the value of x_2 also when $B = 760$ mm., hence,

$$\begin{aligned} -400 &= 4(1{,}000 \log 760 - 1{,}000 \log B_0) \\ &= 4(2880.8 - 1{,}000 \log B_0) \\ 1{,}000 \log B_0 &= 2880.8 + 100 \\ &= 2980.8 \end{aligned}$$

If the decimal is dropped, the number will be more convenient to handle and the effect on the chart will be negligible. The value of x_3 thus becomes $-0.02\ y_3$ $(1{,}000 \log b - 2{,}980)$.

There is the further characteristic of this diagram that lines drawn through points of equal pressure on the various scales are *straight lines passing through the origin*, and that the value of the pressure along the y axis on all the scales is that which corresponds to $\log B_0 = \log b_0 = 2.98$, or $B_0 = b_0 = 955$ mm. The various pressure scales are logarithmic, and the change of the size of the units is such that pressures appropriate to the various levels lie above each other.

It will be noted that all the values of these coordinates except y_1 are negative. This is because the diagram is located in the third quadrant, to the left of the y axis and below the x axis.

The diagram is presented schematically in figure 1. To the left of the origin on the x axis is the temperature scale. Parallel to the temperature scale, but at a certain distance c below it, is the sea-level pressure scale B. At intermediate points, computed from the equation for y_3 lie other parallel scales, showing pressures at various levels above sea level. The values measured on the temperature scale are called x_1, on the sea-level scale are x_2, and on the intermediate scale, x_3. (Two intermediate scales are shown here, x'_3 and x''_3, at distances of y'_3 and y''_3 below the x axis, corresponding to altitudes h' and h'' above sea level.) At either end of the horizontal scales are drawn vertical scales graduated to read altitudes above sea level. The property that points of equal pressure on the various scales may be joined by a straight line passing through the origin is illustrated by the two lines $B'b'_2O$ and $b'_1b''O$, in which $B' = b'_2$ and $b'_1 = b''$. The method of using the chart is shown by the straight line AB. If any two of the three elements (1) mean temperature of the air column, (2) station pressure, or (3) sea-level pressure, are known, it is possible by joining them with a straight line to read directly the third on the proper scale.

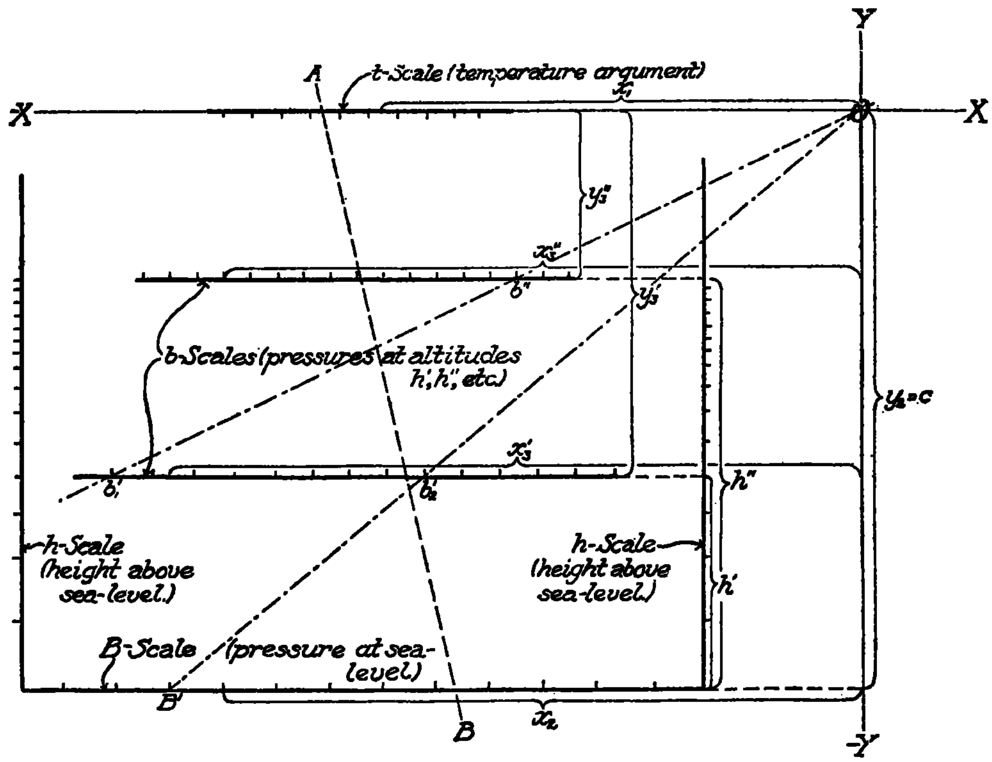

FIG. 1.—Schematic diagram of pressure reduction chart.

The equations for the preparation of the diagram as given by Schwerdt and Loebe, presented above, are convenient for deep layers in the atmosphere—from sea level to 10 kilometers or more. It is often desirable to have the diagram expanded vertically so that shorter air columns can be used with greater accuracy. This is only a matter of deciding upon the values of c and k, and of making appropriate transposition of the numerical factor between the denominator of x_1 and y_3, as discussed above. The following values have been found to be convenient for a diagram, which includes only the lower two kilometers of the atmosphere (see fig. 2):

$x_1=-533.333/(1+\alpha t)$	$y_1=0$
$x_2=8{,}000\ (\log B-3.07)$	$y_2=-400$
$x_3=-20\ y_3\ (\log b-3.07)$	$y_3=-400/(1+0.0008145h)$

kinds of units. In figure 2 and in the coordinate just given, the units are millibars and the convenient value is 3,070. This figure shows the form of the finished chart with a line drawn upon it as an example. Owing to its small size and possible distortion in reproduction, this one is not suitable for practical use by the reader.

Tables 1 to 4 give the data for the plotting of a chart such as that here given. While these cover only this special form of chart, they are considered worthy of inclusion in view of the considerable amount of computation involved and the consequent saving of time to anyone who desires to construct such a diagram for his own use. The size of the chart depends, of course, upon the

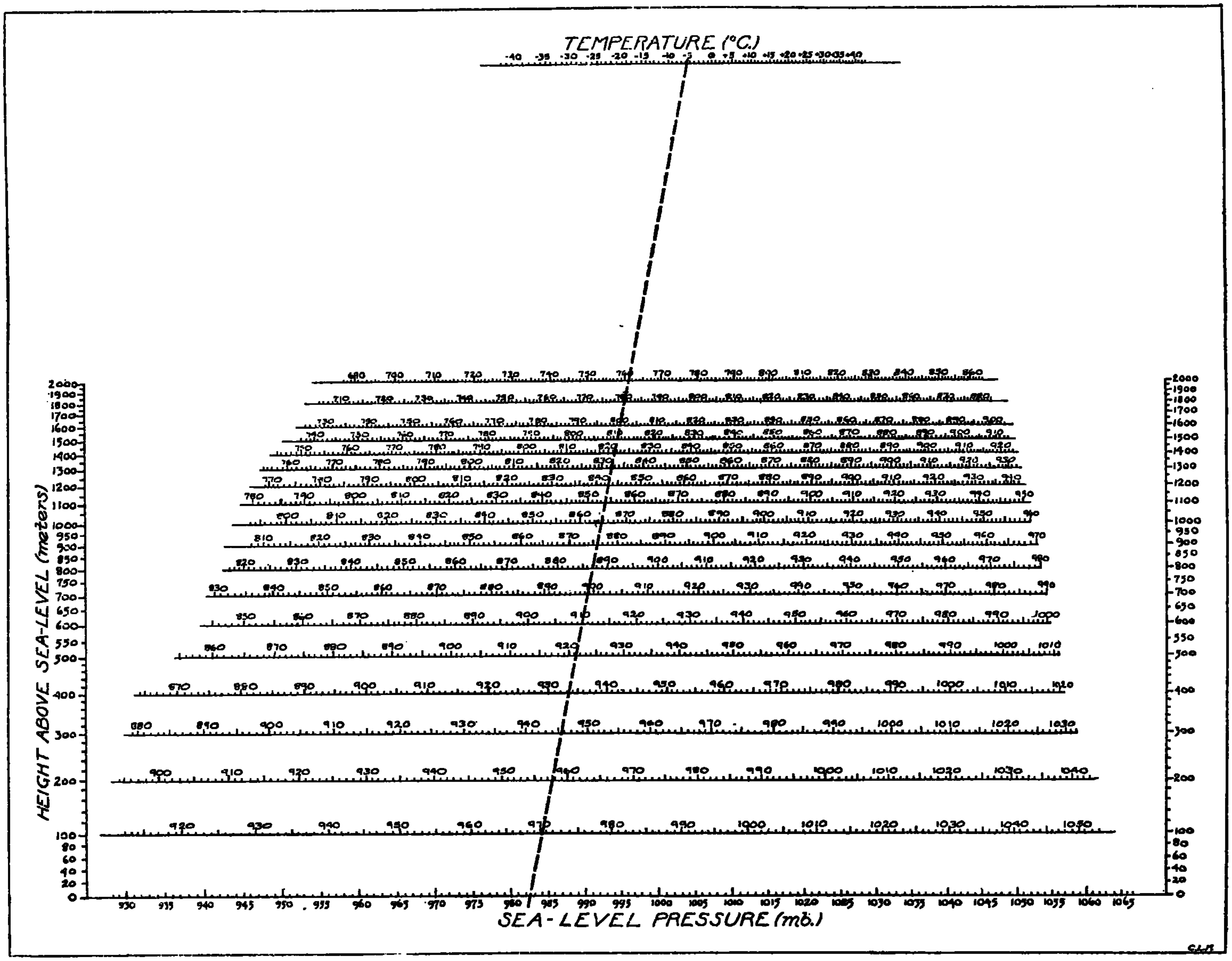

FIG. 2.—Pressure reduction chart. NOTE.—This chart is not to be used for actual pressure reductions, owing to its small size, the slight inaccuracies in the units divisions of the scales (every 5 mb. are accurately located), and possible distortion in reproduction.

A very important warning should be made regarding the numerical value of $\log b_0$ and $\log B_0$, for this is different with different pressure units for diagrams of the same size. For example, as was pointed out above, the authors use the value 2,980, and their pressure scales are in millimeters of mercury. If, on the other hand, the pressure units were millibars, this value would be 3,218. It will be readily seen that this is merely the result of the difference in the logarithm of equal pressures in the two

size of the units of length chosen; but to meet any demands for accuracy that are likely to arise, the tables have been given to two decimal places. Too much emphasis can not be placed upon the necessity for extraordinary care and precision in the location of the points. This diligence will yield a reward in the accuracy of the chart, which, for ordinary work, will be quite as satisfactory as the hypsometric formula and much simpler and quicker to use.

TABLE 1.—*Values of the t-scale* (x_1).

[$x_1 = -533.333/1+\alpha t.$]

t	x_1	t	x_1
°C.		°C.	
40	−465.06	0	−533.33
35	−472.62	− 5	−543.30
30	−480.44	−10	−553.65
25	−488.51	−15	−564.40
20	−496.86	−20	−575.58
15	−505.50	−25	−587.21
10	−514.45	−30	−599.32
5	−523.72	−35	−611.94
0	−533.33	−40	−625.10

TABLE 2.—*Values of the B-scale* (x_2).

[$x_2 = 8(1000 \log B - 3070).$]

B	x_2	B	x_2
mb.		mb.	
930	−812.06	1,000	−560.00
935	−793.47	1,005	−542.64
940	−774.96	1,010	−525.36
945	−756.56	1,015	−508.16
950	−738.24	1,020	−491.08
955	−720.01	1,025	−474.08
960	−701.88	1,030	−457.18
965	−683.84	1,035	−440.38
970	−665.89	1,040	−423.66
975	−648.03	1,045	−407.02
980	−630.25	1,050	−390.46
985	−612.55	1,055	−373.98
990	−594.95	1,060	−357.57
995	−577.45	1,065	−341.25

TABLE 3.—*Values of the h-scale* (y_3).

[$y_3 = -400/1+0.0008145\ h.$]

h	y_3	h	y_3
m. above s. l.		m. above s. l.	
0	−400.00	650	−261.54
20	−393.59	700	−254.75
40	−387.38	750	−248.31
60	−381.36	800	−242.19
80	−375.53	850	−236.36
100	−369.87	900	−230.81
120	−364.38	950	−225.51
140	−359.06	1,000	−220.45
160	−353.88	1,050	−215.61
180	−348.85	1,100	−210.98
200	−343.97	1,150	−206.54
220	−339.22	1,200	−202.28
240	−334.59	1,250	−198.20
260	−330.10	1,300	−194.28
280	−325.72	1,350	−190.52
300	−321.45	1,400	−186.89
320	−317.30	1,450	−183.40
340	−313.25	1,500	−180.04
360	−309.30	1,550	−176.80
380	−305.46	1,600	−173.67
400	−301.70	1,650	−170.65
420	−298.04	1,700	−167.74
440	−294.47	1,750	−164.92
460	−290.98	1,800	−162.20
480	−287.57	1,850	−159.56
500	−284.24	1,900	−157.01
550	−276.25	1,950	−154.54
600	−268.69	2,000	−152.15

TABLE 4.—*Values of the b-scale* (x_3).

[$x_3 = y_3\ (1{,}000 \log b - 3{,}070)/-50.$]

h=2,000 m.		h=1,800 m.		h=1,600 m.		h=1,400 m.		h=1,200 m.	
b	x_3	b	x_3	b	x_3	b	x_3	b	x_3
Mb.		Mb.		Mb.		Mb.		Mb.	
690	−703.39	705	−719.56	730	−717.79	745	−739.54	780	−719.80
695	−693.85	710	−709.60	735	−707.50	750	−728.68	785	−708.58
700	−684.38	715	−699.71	740	−697.27	755	−717.89	790	−697.42
705	−674.97	720	−689.90	745	−687.11	760	−707.18	795	−686.34
710	−665.63	725	−680.14	750	−677.02	765	−696.54	800	−675.32
715	−656.36	730	−670.46	755	−667.00	770	−685.96	805	−664.37
720	−647.15	735	−660.84	760	−657.04	775	−675.45	810	−653.49
725	−638.00	740	−651.29	765	−647.16	780	−665.01	815	−642.68
730	−628.92	745	−641.81	770	−637.33	785	−654.64	820	−631.93
735	−619.90	750	−632.38	775	−627.56	790	−644.33	825	−621.25
740	−610.94	755	−623.02	780	−617.86	795	−634.09	830	−610.63
745	−602.04	760	−613.72	785	−608.23	800	−623.91	835	−599.67
750	−593.20	765	−604.48	790	−598.65	805	−613.79	840	−589.59
755	−584.42	770	−595.30	795	−589.14	810	−603.74	845	−579.16
760	−575.69	775	−586.18	800	−579.68	815	−593.75	850	−568.79
765	−567.03	780	−577.12	805	−570.28	820	−583.82	855	−558.49
770	−558.41	785	−568.12	810	−560.94	825	−573.96	860	−548.24
775	−549.86	790	−559.18	815	−551.66	830	−564.15	865	−538.05
780	−541.36	795	−550.29	820	−542.43	835	−554.02	870	−527.93
785	−532.92	800	−541.46	825	−533.26	840	−544.71	875	−517.86
790	−524.53	805	−532.68	830	−524.15	845	−535.07	880	−507.84
795	−516.19	810	−523.95	835	−514.74	850	−525.49	885	−497.89
800	−507.91	815	−515.28	840	−506.09	855	−515.97	890	−487.99
805	−499.67	820	−506.67	845	−497.14	860	−506.50	895	−478.14
810	−491.49	825	−498.10	850	−488.24	865	−497.09	900	−468.36
815	−483.36	830	−489.59	855	−479.39	870	−487.74	905	−458.62
820	−475.27	835	−480.80	860	−470.60	875	−478.43	910	−448.94
825	−467.24	840	−472.72	865	−461.85	880	−469.18	915	−439.31
830	−459.26	845	−464.36	870	−453.16	885	−459.99	920	−429.73
835	−451.01	850	−456.04	875	−444.52	890	−450.84	925	−420.21
840	−443.43	855	−447.78	880	−435.92	895	−441.75	930	−410.74
845	−435.58	860	−439.56	885	−427.38	900	−432.70		
850	−427.79	865	−431.40	890	−418.88	905	−423.71		
855	−420.04			895	−410.43	910	−414.77		
				900	−402.03				

h=1,000 m.		h=800 m.		h=600 m.		h=400 m.		h=200 m.	
b	x_3	b	x_3	b	x_3	b	x_3	b	x_3
Mb.		Mb.		Mb.		Mb.		Mb.	
795	−747.91	825	−743.78	850	−755.48	865	−802.42	900	−796.30
800	−735.91	830	−731.07	855	−741.80	870	−787.32	905	−779.74
805	−723.98	835	−717.94	860	−728.18	875	−772.30	910	−763.29
810	−712.12	840	−705.87	865	−714.66	880	−757.37	915	−746.92
815	−700.33	845	−693.39	870	−701.21	885	−742.53	920	−730.63
820	−688.62	850	−680.97	875	−687.83	890	−727.76	925	−714.44
825	−676.98	855	−668.64	880	−674.53	895	−713.08	930	−698.34
830	−665.42	860	−656.37	885	−661.31	900	−698.48	935	−682.31
835	−653.47	865	−644.17	890	−648.16	905	−683.96	940	−666.38
840	−642.48	870	−632.05	895	−635.08	910	−669.53	945	−650.53
845	−631.12	875	−619.99	900	−622.08	915	−655.17	950	−634.77
850	−619.82	880	−608.00	905	−609.15	920	−640.88	955	−619.09
855	−608.59	885	−596.09	910	−596.20	925	−626.70	960	−603.49
860	−597.42	890	−584.23	915	−583.50	930	−612.55	965	−587.97
865	−586.33	895	−572.45	920	−570.78	935	−598.50	970	−572.52
870	−575.29	900	−560.73	925	−558.13	940	−584.53	975	−557.16
875	−564.32	905	−549.07	930	−545.55	945	−570.62	980	−541.89
880	−553.40	910	−537.49	935	−533.04	950	−556.79	985	−526.68
885	−542.56	915	−525.96	940	−520.59	955	−543.04	990	−511.56
890	−531.77	920	−514.49	945	−508.21	960	−529.36	995	−496.51
895	−521.04	925	−503.09	950	−495.89	965	−515.74	1,000	−481.53
900	−510.38	930	−491.75	955	−483.64	970	−502.20	1,005	−466.63
905	−499.76	935	−480.47	960	−471.46	975	−488.72	1,010	−451.81
910	−489.22	940	−469.25	965	−459.33	980	−475.32	1,015	−437.05
915	−478.72	945	−458.09	970	−447.27	985	−461.99	1,020	−422.37
920	−468.29	950	−446.98	975	−435.27	990	−448.72	1,025	−407.76
925	−457.91	955	−435.94	980	−423.32	995	−435.52	1,030	−393.22
930	−447.59	960	−424.96	985	−411.46	1,000	−422.38	1,035	−378.76
935	−437.32	965	−414.03	990	−399.64	1,005	−409.31	1,040	−364.36
940	−427.11	970	−403.16	995	−387.88	1,010	−396.31		
945	−416.95	975	−392.34	1,000	−376.18	1,015	−383.36		
950	−406.84								